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A Review on Mesh Generation Algorithms

Lokesh Singh
Mechanical Engineering Department, Lokeshsingh@gmx.com
GD Rungta College of Engineering & Technology, Bhilai, 490023, India

Abstract
Meshing is a process of spatial decomposition. With the required topology and geometric constraints a physical space is decomposed in small parts (called as elements). Mesh generation is a relatively young and multidisciplinary science, it requires expertise in the areas of numerical analysis, meshing algorithms, geometric design, computational geometry, computational physics, scientific visualization and software engineering to create a mesh tool. Meshing can be used for a wide variety of applications; the principal application of interest is the finite element method. In this study, a detail of mesh generation algorithms are discussed for both two dimensional Triangular and Rectangular elements.
Keywords: Meshing algorithms, Delaunay triangulation, Paving, Plastering.

1. Introduction

Meshing is a process of spatial decomposition. With the required topology and geometric constraints a physical space is decomposed in small parts (called as elements), it may also be termed as the process of breaking up a physical domain into smaller sub-domains (Lu et al. 2001). Mesh generation is a relatively young and multidisciplinary science, it requires expertise in the areas of numerical analysis, meshing algorithms, geometric design, computational geometry, computational physics, scientific visualization and software engineering to create a mesh tool. Considerable progress has been made in meshing technology; there are wide variety of tools available commercially and open sources (Hansen and Owen 2008). Over the last 30 years finite element analysis has evolved hand in hand with the ever increasing hardware capability. With the advent of the fast advanced solvers, meshing has been and remains to be significant bottleneck in fully exploiting FEA's great potential (Canann et al. 1997). Even with the fully automatic mesh generators there are many cases where the solution time can be less than the meshing time. Although meshing can be used for a wide variety of applications, the principal application of interest is the finite element method. Surface domains may be subdivided into triangular or quadrilateral shapes, while volumes may be subdivided primarily into tetrahedral or hexahedral shapes. Meshing algorithms ideally define the shape and distribution of the elements. Mesh generation is usually considered as the pre-processing step for numerical computational techniques.

2. Meshing Algorithms

A key step of the finite element method for numerical computation is mesh generation algorithms. A given domain (such as a polygon or polyhedron; more realistic versions of the problem allow curved domain boundaries) is to be partitioned it into simpler "elements". There should be few elements, but some portions of the domain may need small elements so that the computation is more accurate there. All elements should be "well shaped" (which means different things in different situations, but generally involves bounds on the angles or aspect ratio of the elements).

2.1 Quadrilateral or Hexahedral Mesh

Some of the methods of generating quadrilateral / hexahedral meshes are as follows

Grid-Based Method

The grid based approach was first proposed by Schneider's (1996) (R. Schneiders 1996). The mesh method is summarized in a few steps:

(a) A user define grid is fitted on 2 & 3 Dimensional (D) object.
(b) Generates quad/ hex elements on the interior of the object.

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(c) Defining some patterns (i.e. grid contain boundary of object and needs to convert in element) for boundary elements.
(d) Applying in the boundary intersecting grid and form a boundary element.
(e) Finally quadrilateral mesh model generated.

The procedure is illustrated in Figure 1

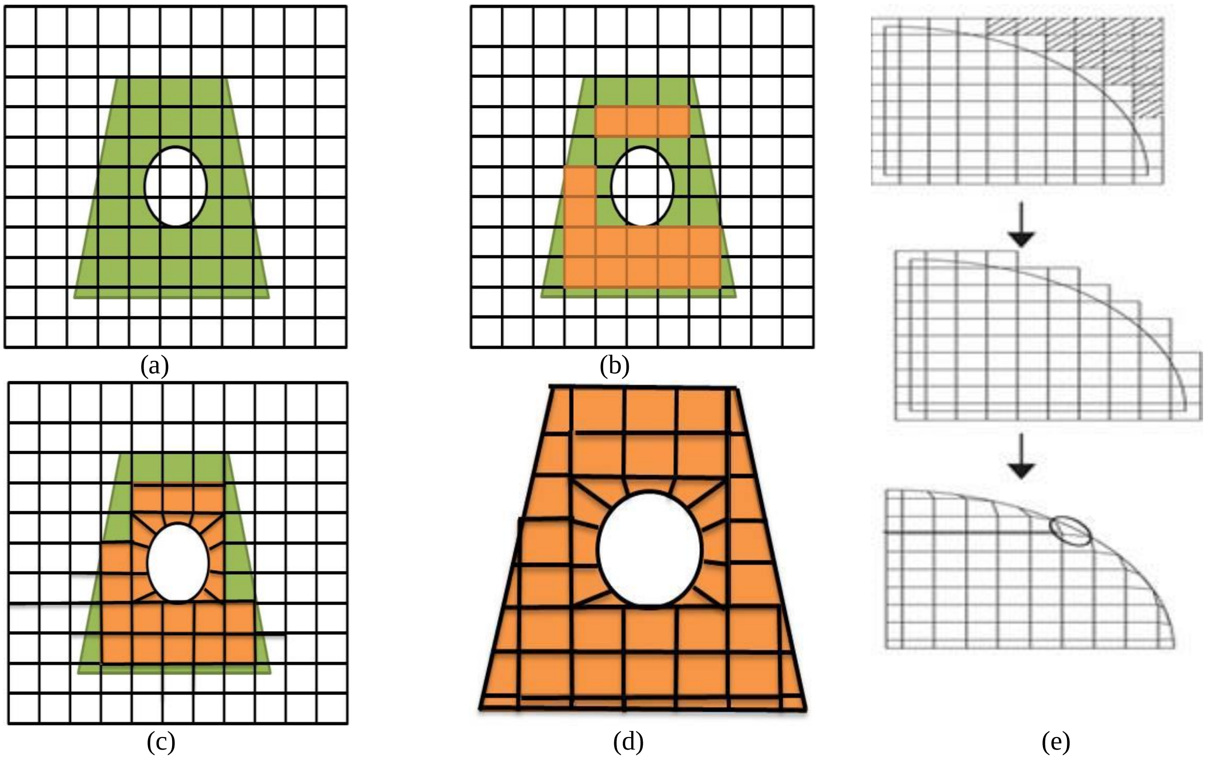

Figure1: Grid-based method (a) Object covered with grid (b) Interior complete quad element (c) Using pattern for making incomplete to complete quad element (d) Complete mesh generation (e) Standard pattern

Medial Surface

Medial axis method is proposed by Tam 1991(Robert Schneiders 2000), it involves an initial decomposition of the volumes and the method is direct extension for quad meshing. The method had few steps for mesh generation:

(a) Considering a 2D object with hole as shown in Figure 2(a), the steps are as follows.
(b) Roll a maximal circle through the model and the centre of circle traces the medial object.
(c) Medial object is used as a tool for automatically decompose the model in to simple meshable region.
(d) Series of templates for the expected topology off the region are formed by the medial axis method to fill the area with quad element.

The procedure is illustrated in Figure 2.

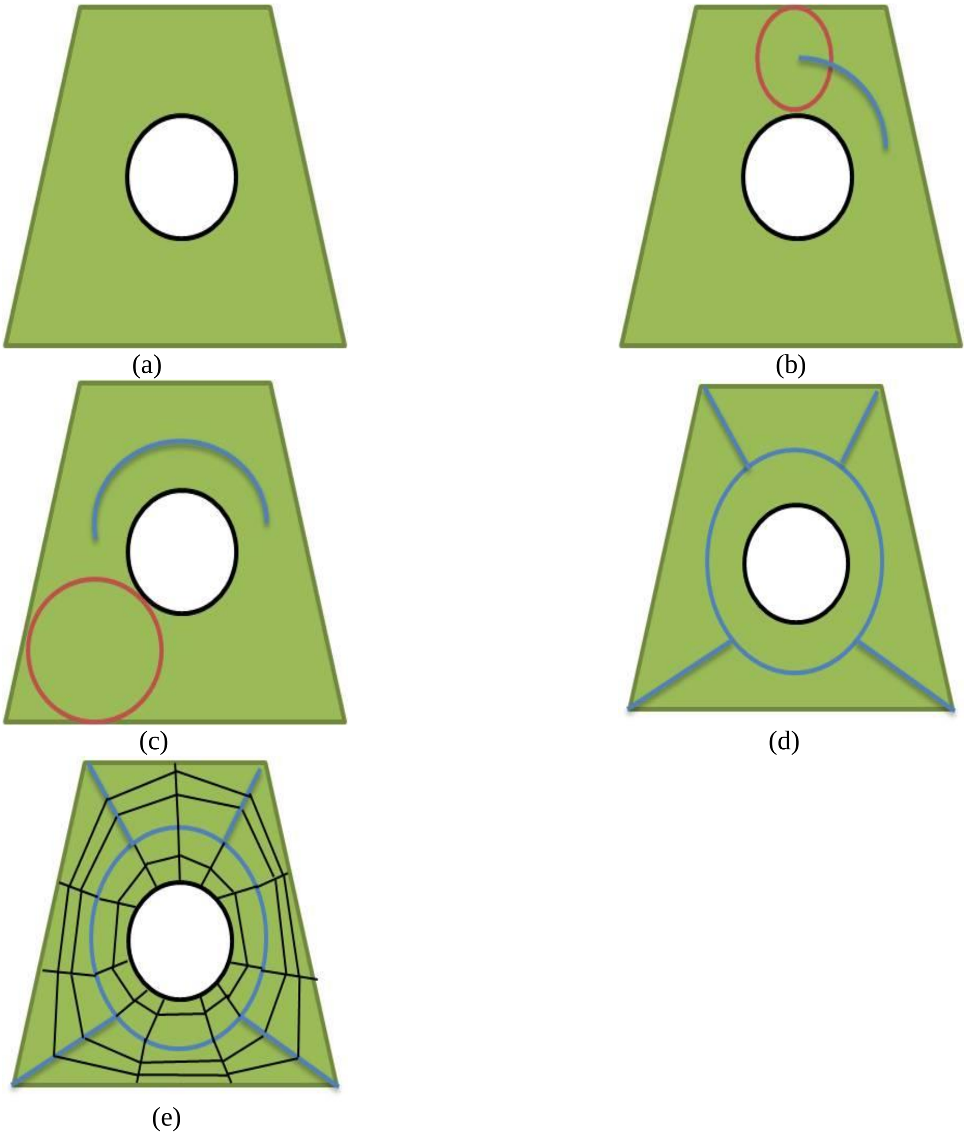

Figure 2: Medial axis: method (a) Object with hole (b) Maximum circle roll with in an object (c) Maximum circle roll with in an object (d) Centre tracing the medial object (e) Meshing of object.

Plastering method

Plastering is proposed by Scott A Cannon in 1991(Owen 1998). It is a process in which elements are placed starting with the boundaries and advancing towards the centre of the volume. The steps of this method can be summarized as follows:

(a) Initially a 3D object is taken.
(b) At boundary one hexahedral element is placed.
(c) Individual hexahedral elements row by row and element by element are projected towards the interior of the volume to form hexahedral meshing.
(d) The process is repeated until mesh generation is completed.

The procedure is illustrated in Figure 3.

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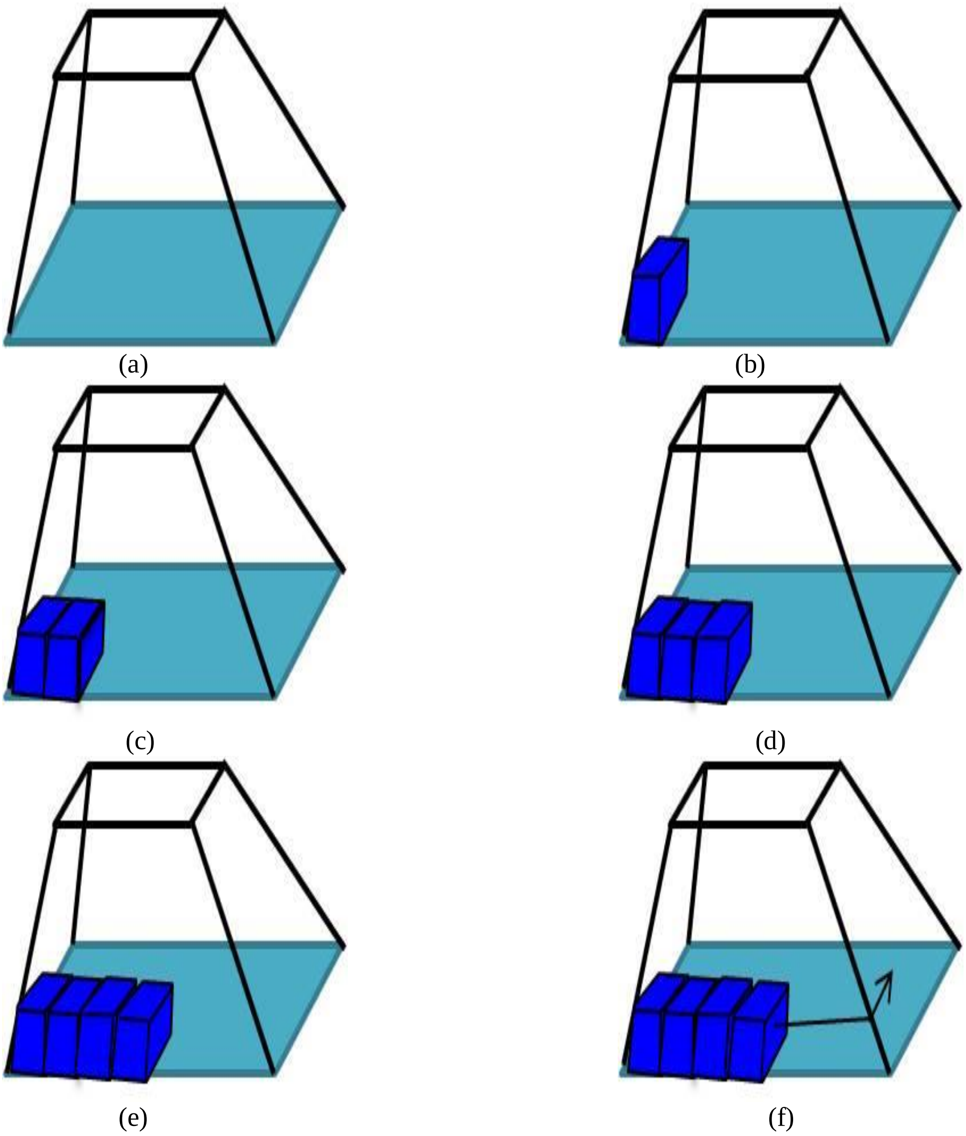

Figure 3: Plastering method (a) 3D object (b) Single element in a row(c) Two element in a same row (d) Three elements in same row (e) Four elements in same row (f) Extension of element in a different row.

Whisker Weaving

Whisker weaving, which was first introduced by T. J. Tautges and Ted Blacker in 1995 (Ted Blacker et al. 1997), is based on the concept of the spatial twist continuum (STC). The STC is described as the dual of the hexahedral mesh, represented by an arrangement of intersecting surfaces, which bisect hexahedral elements in each direction as shown in Figure 4 (Folwell and Mitchell 1999). The whisker weaving algorithm can be explained as in the following steps:

The principle behind this method is to first construct the STC or dual of the hex mesh. With a complete STC, the hex elements can then be fitted into the volume using the STC as a guide. This is done by beginning with a topological representation of the loops formed by the intersection of the twist planes with the surface. The loops can be easily determined from an initial quad mesh of the surface. The objective of the algorithm is to determine where the intersections of the twist planes will occur within the volume. Once a valid topological

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representation of the twist planes has been achieved, hexes are then formed inside the volume. One hex is formed wherever three twist planes converge.

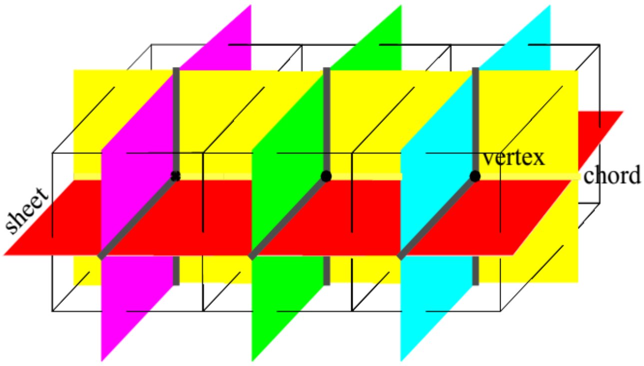


Figure 4: Whisker weaving method

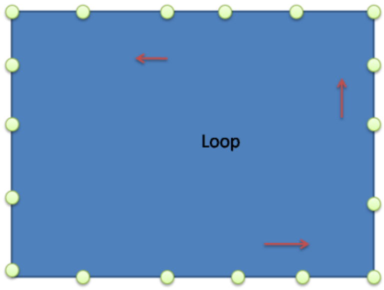
Paving

The paving method was proposed by Ted D. Blacker and Michael B. Stephenson in 1991(Blacker and Stephenson 1991) and the algorithm had a following method to generate a quad mesh as:

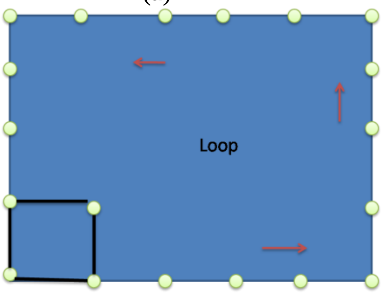
(a) Initially a 2D object is taken.
(b) Inserting a node in the boundary and considering a boundary node as loop.
(c) A quadrilateral element is inserted and a row of elements is formed, the row of element is placed around the boundary nodes.
(d) Again this same procedure adopt for next rows.
(e) Finally quad mesh model is formed.

The procedure is illustrated in Figure 5.

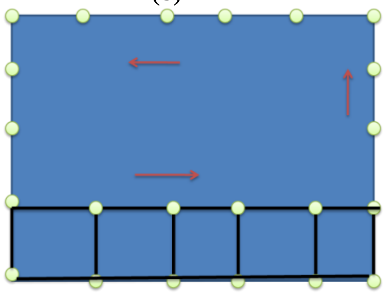
(a)



(b)



(c)



(d)

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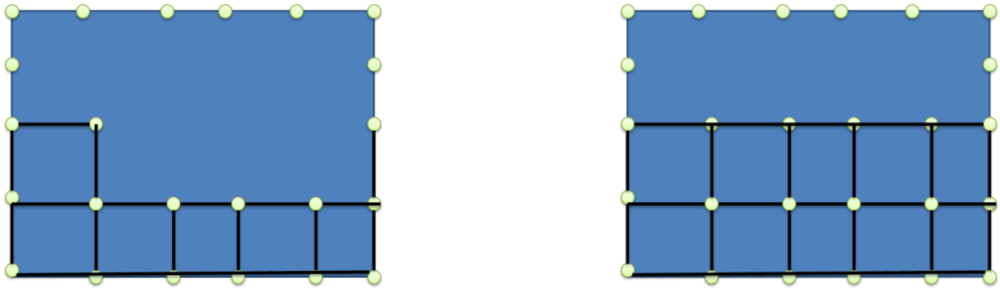


(e) (f)

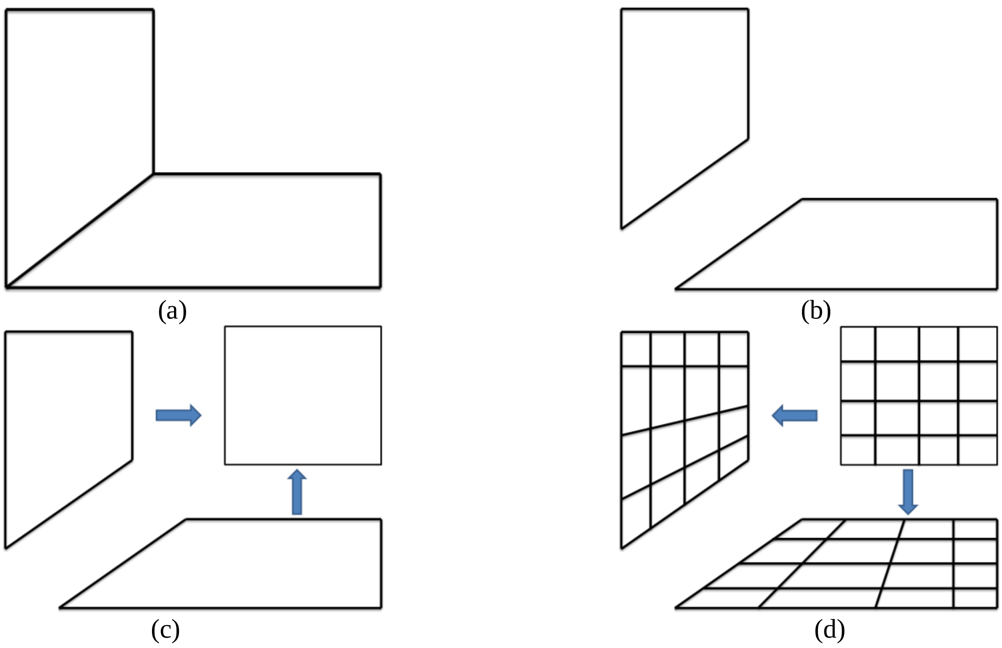
Figure 5: Paving method (a) 2D object (b) Node insertion in boundary and making a loop (c) Single element in a row (d) Complete elements in single row (e) New element second row (f) Complete elements in second row.

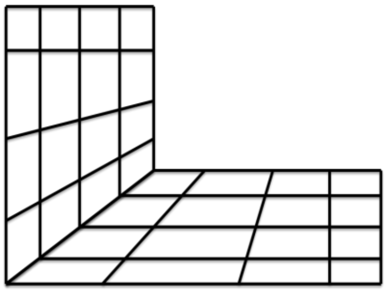
Mapping Mesh Method

Mapped method is proposed by Zienkiewicz and Philips in 1971(N.M. Pitkeathly 2001) and the process of quad mesh generation is as follows:

(a) Initially a 2D object is taken.
(b) 2D object is split in a two parts.
(c) Each part is either a simple 2D rectangular or a square object.
(d) Simple shape object is unit meshed.
(e) Simple shape unit mesh object is mapped in its original form and then join it back to form actual object.

The procedure is illustrated in Figure 6.





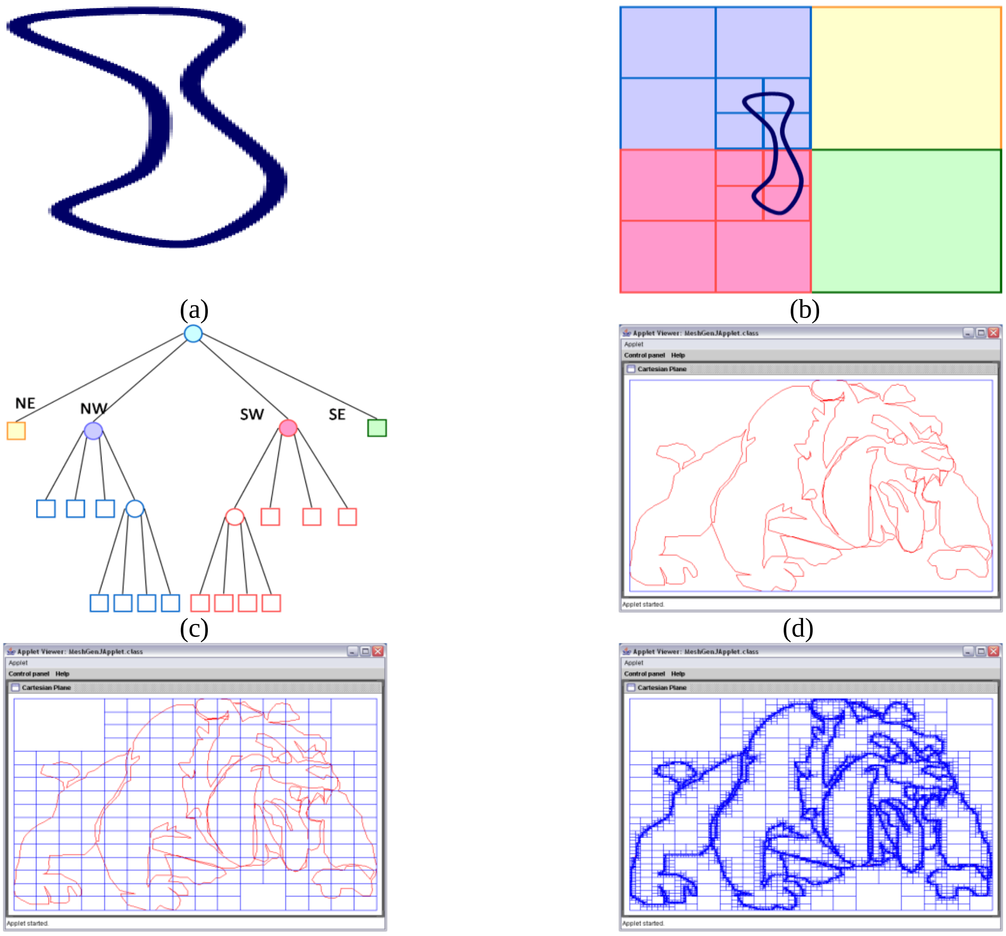
(e)

Figure 6: Mapping and Swapping method (a) 2D object (b) 2D object split (c) Mapping of split object in simple object (d) Quad element in simple object and convert into mapped object (e) Addition of spilt objects.

2.2 Triangle and Tetrahedral Mesh Generation

Quadtree Mesh Method

The Quatree technique was developed in the 1980's by Mark Shepherd's group (Klaas and Shephard 2000). With this method, square containing the geometric model are recursively subdivided until the desired resolution is reached Figure 7(a-c). Figure 7(d-f) shows the steps of two dimensional Quadtree decomposition of a model.



(a) (b) (c) (d) (e) (f)

Figure 7: Quadtree method (a) 2D object (b) Divide of object in rectangular parts (c) Detail tree of divide

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objects (d) Example of object (e) Rectangular division of example (f) Conversion into triangle mesh (Robert Acar, Paul Castillo 2010)

Delaunay Triangulation Method

Delaunay based mesh generation methods were introduced by various authors, (Hermeline, 1980; Watson, 1981), (Legrand et al. 2000). In mathematics and computational geometry, A Delaunay triangulation for a set P of points in the plane is triangulation DT (P) such that no points in P are inside the circum circle of any triangles in DT (P), (Figure 8)

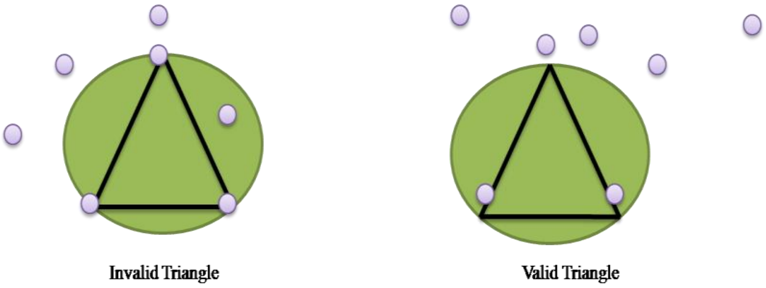


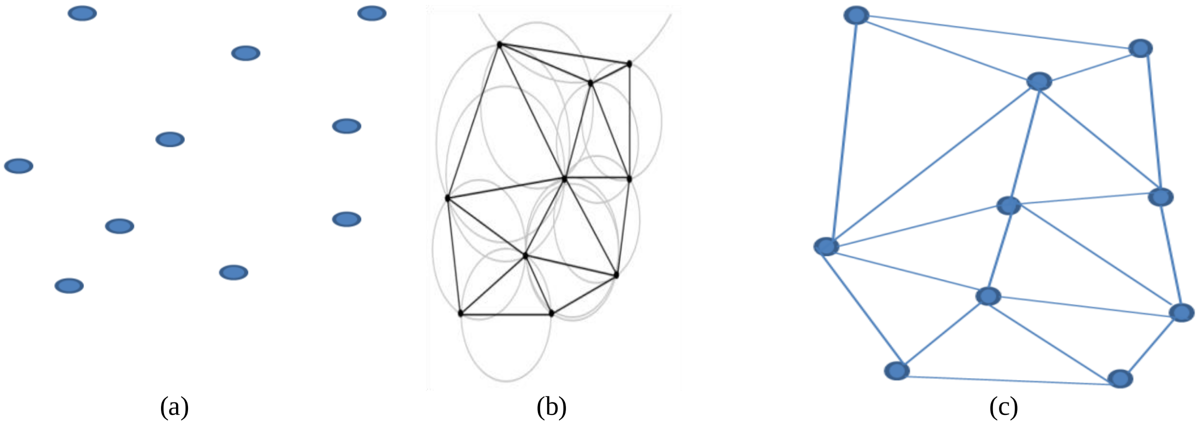
Figure 8: Rule for Delaunay triangulation

The steps of construction Delaunay triangulation are as follows:

(a) Let there be some coordinate points or nodes in space.
(b) The condition of valid or invalid triangle is tested in every three points and finds some valid triangle to make a triangular element.
(c) Finally we had triangular mesh model.

The procedure is illustrated in Figure 9.

Delaunay Triangulation maximize the minimum angle of all the angle of triangle in the triangulation, they tends to avoids skinny triangles (i.e. triangle whose height is much greater than its base).



(a) (b) (c)

Figure 9: Delaunay Triangulation method (a) Random points in space (b) Random circumference circle in points (c) Triangulated model.

Advancing Front Method

Another very popular family of triangular and tetrahedral mesh generation algorithms is the advancing front, or moving front method. A pioneering work proposed by George (1971) (A. EI-hamalawi 2004), for two-dimensional geometries. The mesh generation process is explained as following steps:

(a) Let there be a 2D object with a hole.
(b) Insert a inner and outer boundary nodes (node spacing determined by the user)
(c) Insert an edge to connect the nodes.

(d) For starting meshing process select any edge AB and draw perpendicular a midpoint of AB point C (where C is node spacing determined by the user) and make a triangular element.
(e) After one element is generated another edge is selected as AB and make a point C, but if incase any another node let D is coming within the defined radius then ABC element is cancelled and make a element ABD.
(f) This process is repeated until mesh is generated.

The procedure is illustrated in Figure 10.

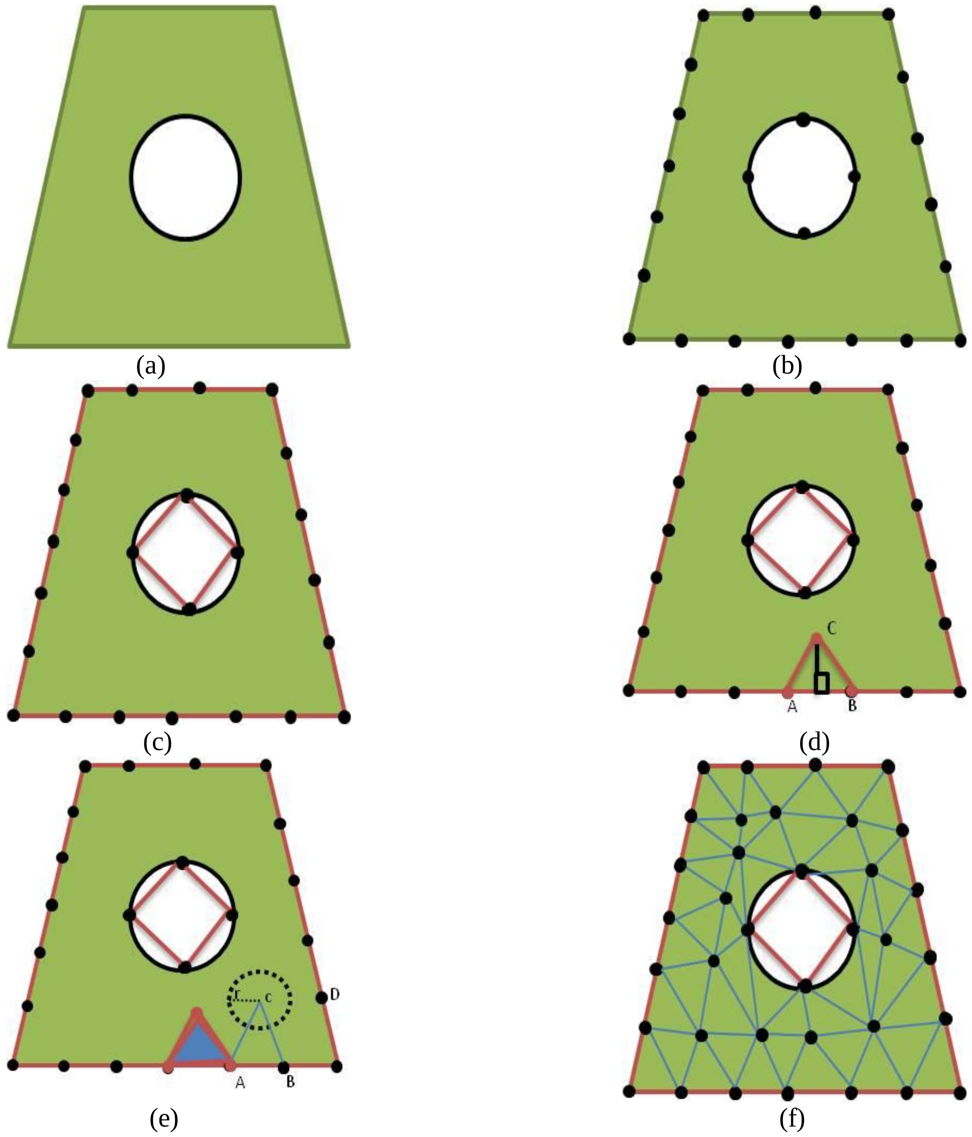


Figure 10: Advancing front method (a)2D object (b) Node insertion in boundary (c) Edge on the boundary (d) Generate one element (e) Apply condition for another element (f) Triangulated model.

Spatial Decomposition Method

The spatial decomposition method is propped by Bykat in 1983 (Ho-Le 1988). The steps for meshing are as follows:

(a) Initially a 2D object is taken.

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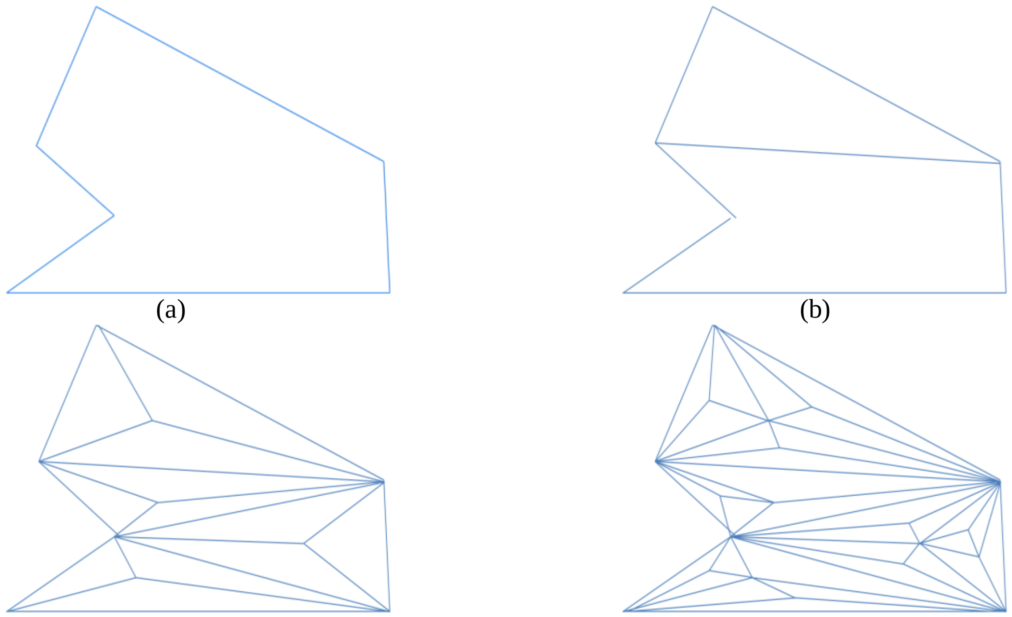
(b) The 2D object is divided in few parts then again it is divided till to we get the refined triangular mesh The procedure is illustrated in Figure 11.

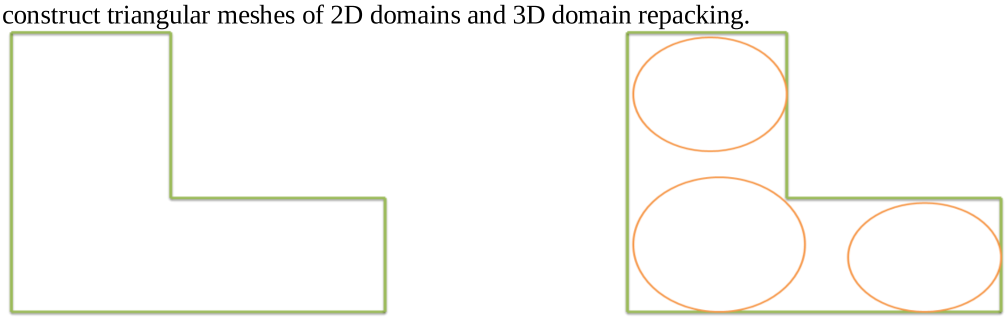


(a) (b) (c) (d)

Figure 11: Spatial decomposition method (a) 2D object (b) Division of object (c) Again division of object (d) Triangulated mesh model.

Sphere Packing Method

The sphere packing method is proposed by Bern, Mitchell and Ruppert in 1990 (Bern and Eppstein 1999). Before constructing a mesh, Figure 12 (a), the domain is filled with circles Figure 12 (b), packed closely together, So that the gaps between them are surrounded by three or four tangent circles. These circles are then used as a framework to construct the mesh, by placing mesh vertices at circle centres, points of tangency, and within each gap Figure 12 (c-d), while using generated points, triangular mesh is generated Figure 12 (e-f). Earlier work by (Shimada and Gossard 1998) also used approximate circle packing and sphere packing to construct triangular meshes of 2D domains and 3D domain repacking.



(a) (b)

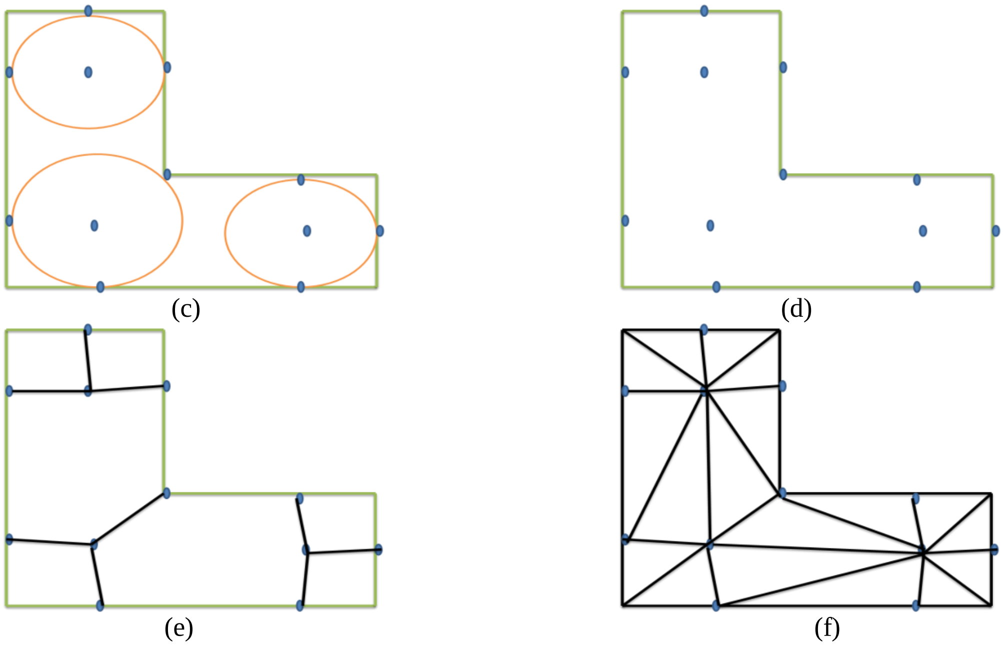


Figure 12: Sphere packing method (a) 2D object (b) Placed maximum circle (c) Node insertion in circle (d) Circle removed (e) Node converts to edge (f) Triangulated mesh model.

The advantage and limitation of the different mesh generation algorithm is summarized in Table no. 1 (Egidi and Maponi 2008), (Chiu et al. 2011), (H. Zhang et al. 2007), (Kraft 1999), (H. Zhang et al. 2007), (Li and Cheng 1996), (Zhu et al. 2014), (Staten et al. 2006), (Folwell and Mitchell 1999).

Table 1: Comparison of Mesh Generation Algorithm

Mesh generation algorithm	**Advantage**	**Disadvantage**
Advancing front method	Mesh boundary sensitive, it produce high quality grid	Computationally expensive, interior voids may be totally unmeshable, not good for sharp angle geometry
Delaunay triangulation	It satisfactory to construct mesh with larger or anisotropic variation of element size, fast run time, good quality of meshing	Large computation time required to obtain the triangulation on an input points set.
Spatial decomposition approach	high quality of element generates directly	Not an automatic process, needs user intervention, produces low quality grid near boundary
Quadtree / octree	It does not need any complex geometric information about input surface	It is not satisfactory to construct a mesh with larger or anisotropic variation of element size
Sphere/circle packing	High quality mesh, robust, it is used to generate nodal point for construction of adaptive and anisotropic mesh	Not sufficient for large scale problem
Grid based approach	Approach is robust and is capable of generating well shaped elements in interior of domain, short development cycle.	Generates poor quality of elements at boundary

Mapped element approach	It is fast , it can control mesh topology and density	Can be used only for domain with simple boundaries
Medial axis transformation	It can be used for subdivision of complex domain into simpler piece for automatic mesh generation and also for generation of simpler idealized model such as shell and beam for stress analysis, works for arbitrarily shape domain	Formulation of the medial axis and recognition of sub regions, when medial axis/surfaces meet.
Paving	It is robust and efficient solution to quad meshing problem, It can handle complex boundary problem	Not computationally efficient, need huge effort for calculating various angles and lengths during mesh generation.
Plastering	It generate well shaped element on the boundary	Not generate good element in interior, not robust and efficient for solution of hexahedral mesh generation
Whisker weaving	It is reliable for large and complex geometry, it produce well shaped element on boundary of domain	Does not generate good element in interior of the domain

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